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18

FINDING OPTIMAL FUEL AND MID-AIR REFUELLING LOCATION REQUIREMENTS FOR C-5A AIRCRAFT

by

CHARLES R. COFFMAN

A project submitted to the Graduate Faculty of

North Carolina State University

in partial fulfillment of the

requirements for the Degree of

Master of Science

DEPARTMENT OF INDUSTRIAL ENGINEERING

RALEIGH

1 9 8 4

APPROVED BY:

Thomas W Reiland

Chairman of Advisory Committee

ABSTRACT

The purpose of this project was to develop a computer program to evaluate the best alternative location and fuel distribution for a large military cargo aircraft when mid-air refuelling may be necessary. The best conditions are considered to be those which minimize the total fuel consumption.

The solution approach used was developed by Abdulrahman Yamani for a dissertation for the University of Florida. Mr. Yamani listed many cases for the overall problem. This program only solves the case with one aircraft, one refuelling, and therefore two legs on the trip for the cargo aircraft.

This program runs on a Commodore 64 microcomputer using standard BASIC language. The map routine is the only section peculiar to this particular machine.

TABLE OF CONTENTS

PROBLEM DESCRIPTION

In the event of a war, earthquake, or other similar disaster anywhere in the world, United States military transport aircraft may respond by delivering equipment, supplies, or personnel to the affected area. These items are loaded into the aircraft taking the physical restrictions of maximum weight limit, type and size of equipment, and others.

The C-5 aircraft is the largest cargo transport in the free world with a maximum gross weight of 760,000 pounds. It has been calculated that 17 C-5s could have accomplished the entire Berlin Airlift. This analysis was based on C-5A characteristics since it is likely to be involved in any large scale airlift operation.

Another physical problem comes up when the distance between the origin base and the destination base is too far for the aircraft to travel without refuelling at a location in between. Bases may be located along the way or it may be more efficient to mid-air refuel using special aircraft. The tanker aircraft flies to a point, meets the transport aircraft, and fuel is transferred from the tanker to the transport while they are still flying.

The objective is to complete the move of the cargo from the origin to the destination with minimum total fuel consumption.

Since we specified the use of the C-5A aircraft, we need to determine:

(1) weight of the cargo load

(2) how much initial fuel to load

(3) transport route

(4) tanker route

(3) and (4) are determined by the refuelling point.

We are only considering the single refuelling case where there are two legs of the trip for the cargo aircraft. An option was added to the program to detect if refuelling was not necessary, neither mid-air nor at a base.

ASSUMPTIONS

(1) The total cargo can be divided any way desired, i.e., we can ignore the fact that the cargo is loaded on pallets and that we can't split pallets. Therefore, the cargo weight can be considered a continuous variable.

(2) No size restriction about the load, i.e., no part of the cargo is too big to fit. This is fairly logical for the C-5A aircraft.

(3) The only limitation on the weight of the aircraft is the gross weight of the aircraft at take-off should not exceed the maximum take-off weight.

$$\text{Aircraft Empty Weight} + \text{Cargo Weight} + \text{Fuel Weight} \leq \text{MTOW}$$

(4) The earth is a perfect sphere with radius R where

R=3920 status miles
=3404 nautical miles

(5) Measure of "distance". The shortest distance between any two points on the surface of a sphere is the shorter great circle arc connecting them. The great circle is the circle in E^3 which has a center at the center of the sphere and is drawn on the surface so that its radius is equal to the radius of the sphere.

(6) The aircraft follows the great circle arc when flying from one point to another.

(7) Weather is assumed to be negligible. (Jet streams or storms have no effect.)

(8) Aerial refuelling takes a negligible amount of time. Therefore, the region in which the refuelling takes place is considered a single point.

NOTATION/DEFINITIONS

(1) Only C-5A aircraft are available

(2) EW = transport Empty Weight

(3) MTOW = transport Maximum Take-Off Weight

(4) F_{max} = transport Maximum Fuel capacity

(5) H_{max} = tanker Maximum Fuel capacity

(6) w = weight of the cargo to be transported

(7) g_o = initial fuel of the cargo aircraft

(8) h_o = initial fuel of the tanker aircraft

(9) $R(g_o,w)$ = Range of the transport when its initial fuel is g_o and its cargo weight is w

(10) $FC(g_o,w,d)$ = Fuel Consumed by the transport when it flies a distance d and its initial fuel is g_o and its cargo weight is w (note: d must be $\leq R(g_o,w)$)

(11) $FN(w,d)$ = exact amount of Fuel Needed to fly a distance d when the cargo weighs w (note: d must be $\leq R(F_{max},0)$)

(12) $L(h_o)$ = range of the tanker when its initial fuel is h_o

(13) $FCT(h_o,d)$ = Fuel Consumed by the Tanker when it flies a distance d and its initial fuel is h_o

(note: d must be $\leq L(h_o)$)

(14) $FNT(d)$ = exact amount of Fuel Needed by the Tanker to fly a distance d (note: d must be $\leq L(H_{max})$)

(15) D_{od} = the shortest distance between the origin base and the destination base measured along the arc of the great circle connecting the two points.

(16) $(\theta_o, \emptyset_o)$ = the coordinates of the origin base in spherical coordinates (longitude and latitude)

$(\theta_d, \emptyset_d)$ = the coordinates of the destination

$(\theta_b, \emptyset_b)$ = the coordinates of the tanker base

$(\theta, \emptyset)$ = any point on a sphere (earth)

(17) $d_1 = d(\theta_o, \emptyset_O, \theta, \emptyset)$ = the distance between the origin and the point $(\theta, \emptyset)$

$d_2 = (\theta_d, \emptyset_d, \theta, \emptyset)$

$d_3 = (\theta_b, \emptyset_b, \theta, \emptyset)$

where $d(\theta_i, \emptyset_i, \theta_j, \emptyset_j)$ = the shortest distance function between the two points $(\theta_i, \emptyset_i)$ and $(\theta_j, \emptyset_j)$ measured along the arc of the great circle connecting the two points.

(18) GW = Gross Weight of the transport

$= EW + g_o + w$

GW_{max} = Maximum Gross Weight of the transport

(19) g = minimum $(F_{max}, GW_{max} - w)$

MEASURE OF DISTANCE

The closest measure of distance models the earth as a sphere. While it is a spheroid, and not a perfect sphere, it is close enough. One travels on a curve when going from one point to another on the surface of the sphere. The curve which has the shortest distance between two points on the surface of the sphere is the great circle. The great circle is the circle in E^3 which has a center at the center of the sphere and is drawn on the surface so that its radius is equal to the radius of the sphere. Therefore, the shortest distance between two points on a sphere is the length of the shorter great circle arc connecting them.

Several methods have been reported in the literature to calculate this distance. All of the methods use two facts:

(1) the great circle distance between two points on a sphere is directly proportional with the radius of the sphere.

(2) the distance between two points on a unit sphere is identical to the angle (measured in Radians) between the two normals of the sphere at these points.

So the distance = $R\alpha$

where R = the radius of the sphere, and

α = the angle between the two normals at these two points (measured in Radians).

The methods differed in the way they found the angle α.

By representing X and Y in spherical coordinates as $(R,\theta_1,\emptyset_1)$ and $(R,\theta_2,\emptyset_2)$, respectively, instead of cartesian coordinates (x_1,x_2,x_3) and (y_1,y_2,y_3), the

$$x_1 = R\sin\emptyset_1\cos\theta_1 \qquad y_1 = R\sin\emptyset_2\cos\theta_2$$

$$x_2 = R\sin\emptyset_1\sin\theta_1 \qquad y_2 = R\sin\emptyset_2\sin\theta_2$$

$$x_3 = R\cos\emptyset_1 \qquad y_3 = R\cos\emptyset_2$$

$\| \cdot \|$ = the Euclidean norm

$\| X \| = \| Y \| = R$

$$\text{COS} = \frac{X \cdot Y}{\| X \| \; \| Y \|}$$

$$\| X \| \cdot \| Y \| = R^2$$

$$X \cdot Y = R^2(\sin\emptyset_1\sin\emptyset_2\cos\theta_1\cos\theta_2 + \sin\emptyset_1\sin\emptyset_2\sin\theta_1\sin\theta_2 + \cos\emptyset_1\cos\emptyset_2)$$

$$= R^2(\sin\emptyset_1\sin\emptyset_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + \cos\emptyset_1\cos\emptyset_2)$$

If we recognize that:

$$\cos(\theta_2 - \theta_1) = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$$

We get:

$$X \cdot Y = R^2(\sin\phi_1\sin\phi_2\cos(\theta_2 - \theta_1) + \cos\phi_1\cos\phi_2)$$
$$\cos \quad = \sin\phi_1\sin\phi_2\cos(\theta_2 - \theta_1) + \cos\phi_1\cos\phi_2$$

where θ = the geographical longitude

ϕ = PI/2 - the latitude = the colatitude

To take advantage of computer routines, we used the following version of this measure:

$$d = R * \arctan\left(\frac{\sqrt{1 - (\sin\phi_1\sin\phi_2\cos(\theta_1-\theta_2) + \cos\phi_1\cos\phi_2)^2}}{\sin\phi_1\sin\phi_2\cos(\theta_1-\theta_2) + \cos\phi_1\cos\phi_2} \right)$$

where θ = longitude * PI/180

ϕ = PI/2 - latitude * PI/180

FUEL CONSUMPTION FUNCTIONS

This section develops the relationships between cargo weight, fuel consumption, distance travelled, and initial fuel.

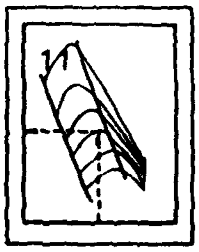
Charts are available from the U.S. Air Force for aircraft performance at different gross weights, speeds, and altitudes. These charts need to be transformed into mathematical formulae before they can be used in a model. An example of this data is shown in Figure 1. Different curves are given for different weights and show the distance travelled per 1,000 lbs of fuel used at a given speed and at the particular gross weight. It is assumed that the transport will operate at the 99% maximum specific range to maximize the distance travelled per unit of fuel burned. The distance in miles per 1,000 lbs of fuel burned is plotted against gross weight GW. The y-coordinate is called MPF(GW) because it is a function of the gross weight. Two kinds of curves to fit those points are tried: linear and quadratic. Both fit well as seen in Figure 2.

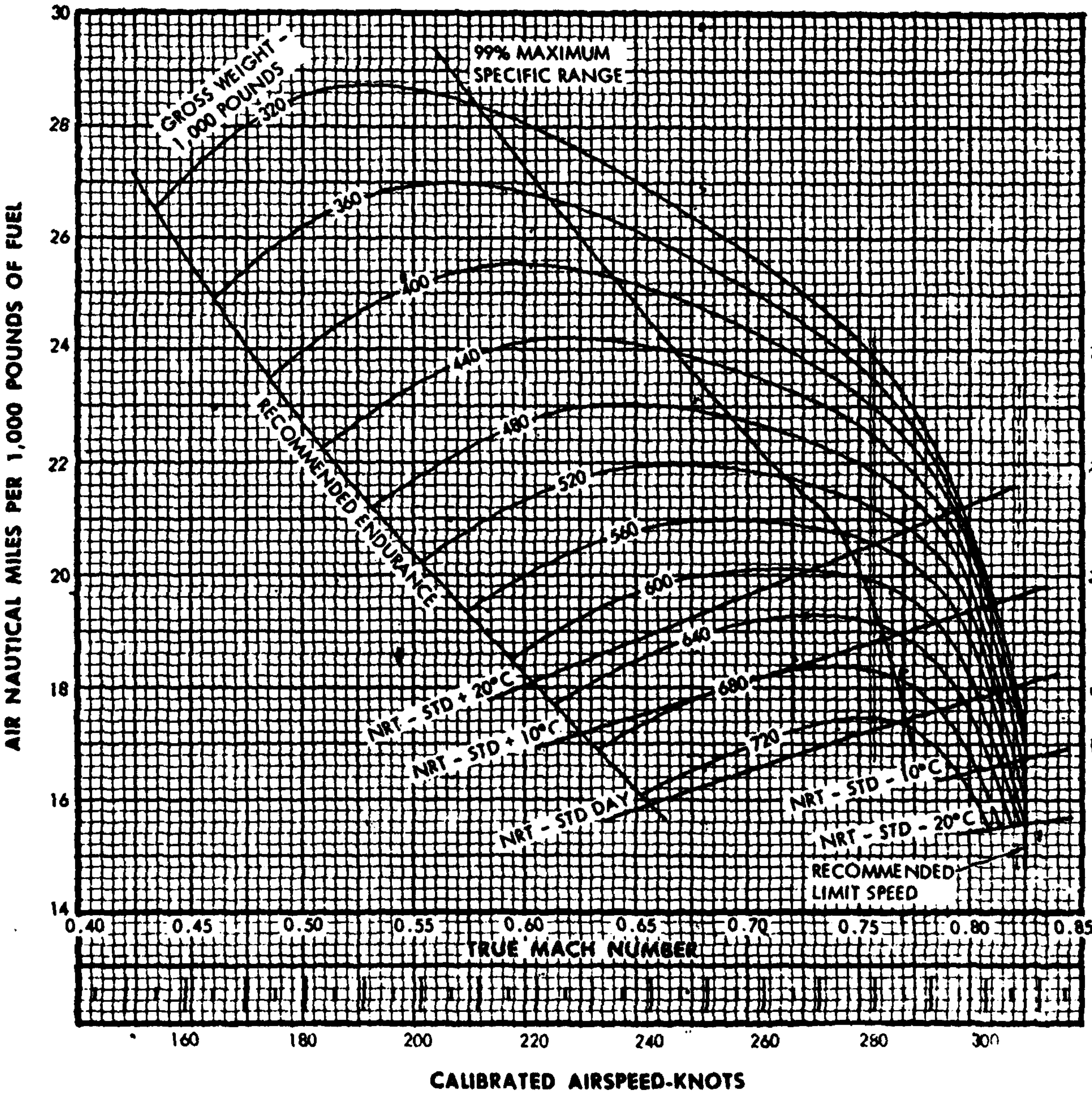
MODEL: C-5A
TF39-GE-1 ENGINES

DATE: APRIL 1972
DATA BASIS:
CATEGORY I/II FLIGHT TEST

SPECIFIC RANGE

4 ENGINES 31,000 FEET





C5A-(1-1)-X5/0-5016

Figure 1. Data for C-5A Aircraft

For the linear fit we have:

$$MPF(GW) = a_o + a_1 GW$$

where:

$$a_o = 36.2829$$

$$a_1 = -0.0270$$

and q = correlation coefficient = -0.9918

For the quadratic fit we have:

$$MPF(GW) = b_o + b_1 GW + b_2 GW^2$$

where:

$$b_o = 43.7616$$

$$b_1 = -0.0576$$

$$b_2 = 2.94 \times 10^{-5}$$

and $q = -0.9983$

We will use the linear fit to find the distance the transport can travel with initial fuel g_o and cargo weight w_o. We can also use it to determine how much fuel is used to travel a distance d with cargo weight w_o and initial fuel g_o.

Note that at any time in flight:

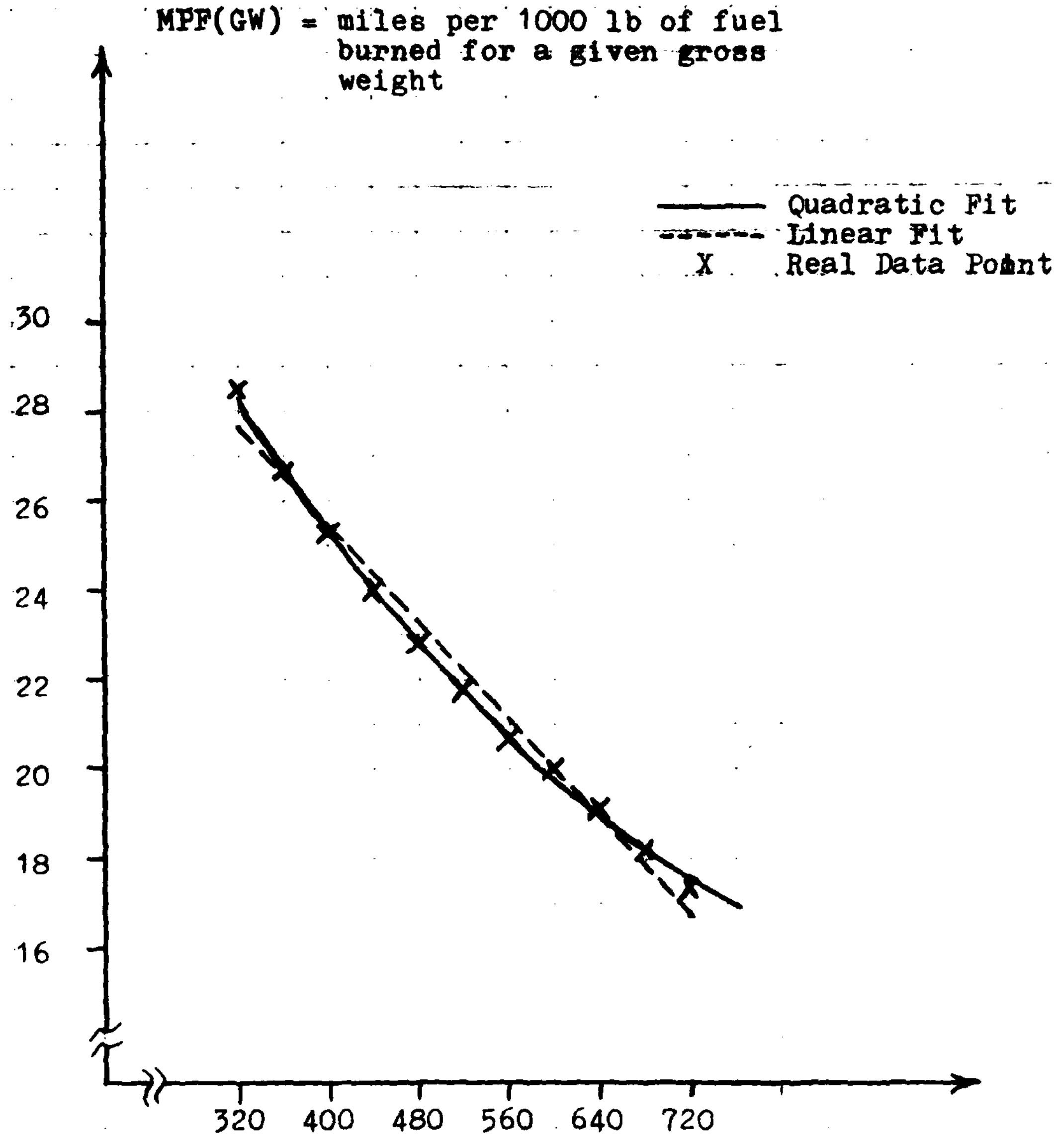
$$GW = EW + w_o + f$$

where f = the instantaneous amount of fuel.

f changes due to burning fuel in flight; therefore, GW changes too. EW and w_o remain constant.

Therefore,

$$dGW = df$$



GW = Gross Weight in 1000 lb

Figure 1. MPF vs. GW

Let $R(g_o, w_o)$ = the range of the transport when its initial fuel is g_o and its cargo weight is w_o.

then $R(g_o, w_o) = \int_{EW+w_o}^{EW+w_o+g_o} MPF(GW)dGW$

$$= \int_{0}^{g_o} MPF(GW)df$$

$$= \int_{0}^{g_o} MPF(EW + w_o + f)df$$

Using the linear fit for MPF,

$$R(g_o, w_o) = (a_o' + a_1 w_o + \frac{a_1}{2} g_o)g_o$$

where $a_o' = a_o + a_1 EW$

Now we must find the fuel consumed when the aircraft flies a distance d ($d \leq R(g_o, w_o)$) with initial fuel g_o and cargo weight w_o.

Let g_f = final amount of fuel left, and

$FC(g_o, w_o, d)$ = fuel consumed when initial fuel is g_o, cargo weight is w_o, and the distance flown is d.

then $FC(g_o, w_o, d) = g_o - g_f$

Therefore, to find FC we need to find g_f, which can be done by solving for g_f in the following equation:

$$d = \int_{g_f}^{g_o} MPF(EW + w_o + f)df$$

For linear fit of MPF we have:

$$d = \int_{g_f}^{g_o} (a_o + a_1(EW + f + w_o))df$$

$$= a_o(g_o - g_f) + a_1(EW + w_o)(g_o - g_f) + \frac{a_1}{2}(g_o^2 - g_f^2)$$

$$0 = \frac{a_1}{2}g_f^2 + (a_o + a_1(EW + w_o))g_f$$

$$- (a_o g_o + a_1(EW + w_o)g_o + \frac{a_1}{2} g_o^2) - d$$

$$\frac{a_1}{2}g_f^2 + ag_f - d' = 0 \quad \text{where} \quad a = a_o + a_1(EW + w_o)$$

This equation is quadratic in g_f. To find the solutions to this equation:

$$g_f = -\frac{a}{a_1} \pm \frac{\sqrt{a^2 + 2a_1 d'}}{a_1}$$

$$g_f = -\frac{a}{a_1} \pm \frac{\sqrt{(a + a_1 g_o)^2 - 2a_1 d}}{a_1}$$

Only the positive solution makes g_f physically possible, so:

$$g_f = -\frac{a}{a_1} \pm \frac{\sqrt{(a + a_1 g_o)^2 - 2a_1 d}}{a_1}$$

Since the fuel consumed = $FC(g_o,w_o,d) = g_o - g_f$, then

$$FC(g_o,w_o,d) = g_o + \frac{a}{a_1} - \frac{\sqrt{(a + a_1 g_o)^2 - 2a_1 d}}{a_1}$$

where $a = a_o + a_1(EW + w_o)$

Let $FN(w_o,d)$ = the exact amount of fuel needed to fly a distance d when the cargo weight is w_o. We need to find FN. For a linear fit of MPF we have:

$$R(g_o,w_o) = d = (a_o' + a_1 w_o + \frac{a_1}{2} g_o)g_o$$

With FN = g_o and solving R for g_o we get:

$$FN(w_o,d) = g_o = -w_o - \frac{a_o'}{a_1} + \frac{\sqrt{(a_o' + a_1 w_o)^2 + 2a_1 d}}{a_1}$$

MODEL AND PROPOSED SOLUTION METHODS

The case where there is one aircraft with predetermined cargo weight and initial fuel, but needs to be refuelled is a subproblem of the much larger problem involving many aircraft and multiple refuelling. It is one of the building blocks to the solution of the larger problem.

This problem can be broken down into two subproblems itself. The first subproblem can be defined as finding the initial fuel needed and other fuel needed to complete the mission given a feasible refuelling point. The second subproblem is to find the location of a refuelling point such that the total fuel used is minimized. Using the results from the first subproblem as input for the second subproblem, and using the results from subproblem two as input for subproblem one in turn, an optimal solution can be found which does not require us to predetermine the initial fuel amount.

Given: EW, MTOW, GW, and FM from the aircraft specifications and w_o from the mission requirements we can determine the allowable fuel loads for take-off and mid-air refuelling.

Let FO = Fuel weight possible to add to aircraft at take-off

$= MTOW - EW - w_o$

GO = Maximum fuel weight possible at mid-air refuelling

$= GW - EW - w_o$

GM = Maximum fuel weight capacity at take-off

= min(FO,FM)

GC = Maximum fuel weight capacity at mid-air refuelling

= min(GO,FM)

Given a suggested refuelling point $(\theta,\emptyset)$, the distance to the respective bases can be determined with:

D1 = distance to the origin base

D2 = distance to the destination base

D3 = distance to the tanker base

The ranges from the bases may be determined by:

$R(g_o,w_o) = R(GM,w_o)$ = Range from origin base

$= R(GC,w_o)$ = Range from destination base

$= \frac{R(H_{max},w_o)}{2}$ = Range from tanker base (1/2 because of return trip)

Now the feasibility of the refuelling point can be determined by comparing the distances to the ranges.

For feasibility:

$D1 \leq R1$, $D2 \leq R2$, $D3 \leq R3$, and the distance from the origin base to the destination base = R1 + R2

Given a feasible initial point we may find the minimum combination of initial fuel, transferred fuel, and tanker fuel.

Min(Total Fuel Cost for Subproblem 1) = $g_o + h_o$ =

$$g_o + \frac{\sqrt{\left(C - \sqrt{(a + a_1 g_o)^2 - 2a_1 D1}\right)^2 + 2a_1 D3}}{a_1} - \frac{a_o'}{a_1}$$

where $C = \sqrt{a^2 + 2a_1 D2} + \sqrt{a_o'^2 + 2a_1 D3}$

We search for the minimum total cost over the range from Low = FN(w_o,D1) to High = GM. A Golden Section Search is used in the program to find this minimum. The accuracy is set at 0.00010. The resulting Total Fuel Cost is checked against previous solutions for change. If there is little change, then the problem is solved. The accuracy is specified in the initial conditions. A large TFC has been defined to caus the problem to cycle at least once. If the change is significant, then go to subproblem two.

Subproblem two involves finding the location that minimizes the sum of fuel costs for the four legs of the mission. The four fuel costs are:

$FN(0,D3)$ = Fuel needed by the tanker to return to base

$FN(w_o,D2)$ = Fuel needed by the transport to get to the destination from the refuelling point

$FN(w_p,D1)$ = Fuel needed by the transport to get to refuelling point with cargo and unused fuel.

where $W_p = w_o + (g_o - FC(g_o,w_o,D1))$

$$\text{and } FC(g_o,w_o,D1) = g_o + w_o + \frac{a_o'}{a_1} - \frac{\sqrt{(a_o' + a_1(w_o + g_o))^2 - 2a_1 D1}}{a_1}$$

$FN(w,D3)$ = Fuel needed by the tanker to get to the refuelling point with net fuel needed to transfer

where

$$w = FN(0,D3) + FN(w_o,D2) - (g_o - FC(g_o,w_o,D1))$$

Subproblem two is subject to the same feasibility constraints as the original problem.

Finding the optimum location involves searching along Θ and Ø in an iterative approach to find the minimum fuel cost. Limits for the different functions were hard to find so the initial feasible point is used as a beginning. Θ and Ø are changed in a systematic manner and whichever yields the lower fuel cost is selected.

(Θ,Ø) yields a cost F2

(Θ+¢,Ø+¢) yields a cost F2'

If (Θ+¢,Ø+¢) not feasible, try another point

If F2' > F2, try another point

If F2' < F2, F2 = F2' and Θ = Θ+¢, Ø = Ø+¢ and try another point in the same direction

If F2' = F2, then reduce ¢ and try another point in same direction

Similarly, when conditions lead us to try another point:

(Θ-¢,Ø+¢), (Θ-¢,Ø-¢), and (Θ+¢,Ø-¢)

In each case, ¢ returns to the original value for the new direction. After one complete pass, ¢ is checked to see if it is below the specified accuracy limit. If it is, then the second subproblem is solved. If not, then reduce the original value of ¢ and make another pass. In each direction, when ties occur, ¢ will reduce to the specified accuracy before trying

another direction. This completes 1 iteration in solving the complete problem.

The process of solving the first subproblem to get initial fuel and then solving the second subproblem to get the optimal location continues until the Total Fuel Cost as defined by subproblem one shows insignificant change.

PROGRAM

The main body of the program consists of defining of constants; input of locations, cargo weight, and accuracy requirements; gosubs for the subproblems; and various output messages.

The transport aircraft and tanker aircraft are assumed to have identical performance charts. Fuel Cost and Fuel Needed subroutines for the tanker would have to be changed if this were not true. The constant C would also have to be changed.

The longitudinal difference between any two points is restricted to 180 degrees and the latitude is restricted to North latitudes. Longitude and latitude are converted to radians using the formulas:
θ = (longitude difference)*PI/180 and
ϕ = PI/2 - latitude*PI/180. The longitude of the origin base is always transformed to 0.

A map of the U.S. and Europe is available at the end of the program which display coordinates from 108°W to 48°E and 0°N to 70°N. This will allow display of most U.S. to Europe or Middle East missions and their return flights.

The longitude of the tanker base should be in the same direction from the origin base as the longitude of the destination base.

The following lines may be hard to read in the program since the printer also printed on the perforations separating the pages:

```
1895 REM  CHECK FOR CORRECT INPUT
32ØØ GOSUB Ø44ØØ : REM  DISTANCE(TA,PA)
64ØØ GR=GM : WR=WO
1Ø4ØØ GØ=XB : SF=(A+A1*XB)↑2-2*A1*D1
153ØØ IF F2=FF THEN TY=1
2262Ø REM  WHICH IS SLIGHTLY DIFFERENT FROM SUBPROBLEM
           ONE BECAUSE OF
28Ø3Ø REM  THIS ROUTINE MUST BE EXPANDED TO ALLOW
           ROUTES ACROSS THE
352ØØ IF (SJ$="E") AND (SL$="W") AND (SN$="E") AND
         (SQ$="E") THEN L6=L1-CA
6Ø533 POKE1991,43:POKE2ØØØ,43:POKE2Ø12,43:POKE2Ø21,43
```

```
100 EW=370.0 : QQ=0 : A1=-0.026785 : AO=36.1459 : MT=700.0
200 GW=760.0 : HM=300.0 : FM=300.0
300 AP=AO+A1*EW : PI=3.14159 : R=3960.*5280./6080. : K=0 : F0=1000000.
310 REM  EW=EMPTY WEIGHT/1000 LBS  MT=MAXIMUM TAKEOFF WEIGHT/1000 LBS
320 REM  GW=MAXIMUM GROSS WEIGHT/1000 LBS  FM=MAXIMUM FUEL CAPACITY/1000 LBS
330 REM  HM=MAXIMUM FUEL CAPACITY/1000 LBS OF TANKER
335 REM  FM=MAXIMUM FUEL CAPACITY/1000 LBS OF TRANSPORT
340 REM  A1=LINEAR SLOPE FROM ENGINE PERFORMANCE CURVES
350 REM  AO=LINEARTR INTERCEPT FROM ENGINE PERFORMANCE CURVES
360 REM  QQ=INITIAL CASE SETTING FOR POSSIBLE LOCATION SCHEMES
370 REM  AP=FUEL REQUIRED TO MOVE EMPTY AIRPLANE  R=SPHERICAL CONSTANT
380 REM  K=ITERATION COUNTER   F0=INITIAL TOTAL COST SET VERY LARGE
400 PRINT "J"
410 PRINT "THIS PROGRAM WILL NOT WORK FOR"
415 PRINT "SOUTH LATITUDES OR FLIGHT PLANS"
420 PRINT "CROSSING THE INTERNATIONAL DATE LINE."
425 PRINT "FUTURE MODIFICATIONS WILL CORRECT"
430 PRINT "THIS SITUATION." : PRINT :PRINT
440 PRINT "ENTER COORDINATES"
450 PRINT "IN DEGREES, THEN DIRECTION"
455 PRINT "EXAMPLE:  20 (RETURN) E (RETURN)"
460 PRINT "FOR A LONGITUDE OF 20 DEGREES EAST"
470 PRINT : PRINT : PRINT
500 INPUT "ORIGIN LONGITUDE";L1 : INPUT "DIRECTION E/W";SJ$:PRINT
550 INPUT "ORIGIN LATITUDE";H1 : INPUT "DIRECTION N/S";SK$:PRINT
600 INPUT "DESTINATION LONGITUDE";L2 : INPUT "DIRECTION E/W";SL$:PRINT
650 INPUT "DESTINATION LATITUDE";H2 : INPUT "DIRECTION N/S";SM$:PRINT
700 INPUT "TANKER BASE LONGITUDE";L3 : INPUT "DIRECTION E/W";SN$:PRINT
750 INPUT "TANKER BASE LATITUDE";H3 : INPUT "DIRECTION N/S";SP$:PRINT
800 INPUT "CARGO WEIGHT (IN POUNDS)";WO
900 PRINT "FUEL COST ACCURACY(TO NEAREST POUND = 1)":
910 PRINT "                  (TO NEAREST 10 POUNDS":
920 INPUT "                  = 10)";E1
930 PRINT
1000 PRINT "LOCATION ACCURACY DESIRED"
1010 PRINT "(TO NEAREST DEGREE = 1 )"
1015 PRINT "(TO NEAREST MINUTE = 2 )"
1020 PRINT "(TO NEAREST SECOND = 3 )"
1030 INPUT "SELECT ACCURACY";RZ:PRINT
1040 WO=WO/1000
1045 E1=E1/1000
1050 IF RZ=1 THEN E2=1*PI/180
1055 IF RZ=2 THEN E2=1/60*PI/180
1060 IF RZ=3 THEN E2=1/360*PI/180
1070 REM  CONVERT INPUTS TO LIKE UNITS FOR CALCULATIONS
1100 FO=MT-EW-WO
1200 G0=GW-EW-WO
1300 IF FM<=FO THEN GM=FM
1350 IF FM>FO THEN GM=FO
1400 IF FM<=G0 THEN GC=FM
1450 IF FM>G0 THEN GC=G0
1500 A=AP+A1*WO
1600 REM  SET LIMITS ON FUEL TO DETERMINE FEASIBILITY
1700 INPUT "PROPOSED LONGITUDE";CA : INPUT "DIRECTION E/W";SQ$:PRINT
1800 INPUT "PROPOSED LATITUDE";CB : INPUT "DIRECTION N/S";SR$:PRINT
1810 PRINT : PRINT
1850 GOSUB 28000  : REM  CONVERT TO RADIANS
1855 PV=0
1860 IF (QQ<1) OR (QQ>12) THEN PV=1
1870 IF SK$="S" OR SM$="S" OR SP$="S" OR SR$="S" THEN PV=1
1880 IF PV=1 THEN PRINT "CHECK INPUTS : PARAMETERS VIOLATED"
1885 IF PV=1 THEN PRINT : PRINT
1890 IF PV=1 THEN GOTO 410
1895 REM  CHECK FOR CORRECT INPUT
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1900 GOSUB 03000 : REM  CHECK FOR FEASIBILITY
1950 IF IN=45 THEN GOTO 410
2000 IF IN<>40 AND IN<>50 THEN GOTO 01700
2005 REM  IF INITIAL LOCATION IS NOT FEASIBLE-TRY AGAIN
2010 IF IN=40 THEN GOSUB 10000
2015 REM  SOLVE SPECIAL CASE WHEN REFUELLING NOT NEEDED
2020 IF IN=40 THEN GOTO 2775
2025 REM  PRINT ANSWER TO SPECIAL CASE
2030 REM
2035 REM
2100 GOSUB 07500 : REM  SOLVE SUBPROBLEM ONE FOR INITIAL FUEL AND TRANSFER FUEL
2110 REM
2115 REM
2200 Z=F-F0 : Z1=ABS(Z) : IF Z1<=E1 THEN GOTO 02700
2210 REM  IF NEW SOLUTION WITHIN ACCURACY LIMITS THEN PRINT SOLUTION
2300 F0=F
2305 PRINT : PRINT
2310 PRINT "TRIAL SOLUTION TO SUBPROBLEM ONE":
2320 PRINT "INITIAL FUEL";G0*1000:
2330 PRINT "TANKER/TRANSFER";H0*1000:
2340 PRINT "TOTAL";F*1000:
2350 PRINT:PRINT
2360 PRINT "WORKING ON PROBLEM TWO":
2370 PRINT :
2380 REM  PRINT SOMETHING SO OPERATOR KNOWS PROGRAM IS WORKING
2385 REM
2390 REM
2400 GOSUB 11700 : REM  SOLVE SUBPROBLEM TWO TO FIND BEST LOCATION
2410 REM
2500 K=K+1
2510 REM  UPDATE ITERATION COUNTER
2520 REM
2530 REM
2600 GOTO 02100
2610 REM  RETURN TO SOLVE SUBPROBLEM ONE FOR NEW LOCATION
2620 REM
2630 REM
2700 GOSUB 50500 : REM  CONVERT ANSWER TO LONGITUDE AND LATITUDE
2750 PRINT "J":PRINT "MINIMUM COST SOLUTION"
2775 PRINT "OPTIMUM REFUELING POINT":PRINT L9;D7$,H8;D8$
2780 PRINT "INITIAL FUEL",G0*1000:
2785 PRINT "TRANSFER FUEL",(F-G0-F7-F3)*1000:
2787 PRINT "TANKER FUEL",(F7+F3)*1000:
2790 PRINT "TOTAL FUEL COST",F*1000:
2800 PRINT "ITERATIONS";K:
2810 PRINT "ORIGIN":PRINT L1;SJ$,H1;SK$ : PRINT "DESTINATION"
2815 PRINT L2;SL$,H2;SM$
2820 PRINT "TANKER BASE":PRINT L3;SN$,H3;SP$
2830 PRINT "CARGO WEIGHT";W0*1000;"LBS"
2840 PRINT:PRINT:PRINT
2845 REM  PRINT FINAL ANSWER AND REPEAT ORIGINAL INFORMATION
2880 PRINT "A MAP OF U.S. TO EUROPE AND WEST ASIA":
2881 PRINT "IS AVAILABLE.  DO YOU WISH TO SEE?" : INPUT "(Y/N)";MP$
2885 IF MP$="Y" THEN GOSUB 60500
2890 REM  MAP DEMONSTRATION AVAILABLE
2895 GOTO 2750
2900 STOP
2910 REM
2920 REM
2930 REM
2940 REM
3000 REM  FEASIBLE
3010 REM  THIS ROUTINE DETERMINES IF THE PROBLEM IS FEASIBLE FOR THE GIVEN
3020 REM  PARAMETERS.
3100 TA=TH : PA=PH
3200 GOSUB 04400 : REM  DISTANCE(TA,PA)
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3210 REM  CALCULATE THE DISTANCE FROM THE TRIAL POINT TO THE THREE BASE POINT
3300 GOSUB 06300 : REM  RANGE
3310 REM  CALCULATE THE RANGE OF AIRCRAFT FROM THE THREE BASE POINTS
3320 WL=WO : DL=D1 : GOSUB 08200 : REM  FNTR(WO,D1)
3375 IN=0
3400 IF (D1>R1) OR (FT>GM) THEN IN=10
3500 IF D2>R2 THEN IN=20
3600 IF D3>R3 THEN IN=30
3610 IF O9<R1 THEN IN=40
3620 IF O9>R1+R2 THEN IN=45
3700 IF IN=0 THEN IN=50
3800 IF IN=10 THEN I$="TOO FAR FROM ORIGIN"
3900 IF IN=20 THEN I$="TOO FAR FROM DESTINATION"
4000 IF IN=30 THEN I$="TOO FAR FROM TANKER BASE"
4010 IF IN=40 THEN I$="REFUELING NOT NEEDED"
4020 IF IN=45 THEN I$="INFEASIBLE-ORIGIN TO DESTINATION TOO FAR"
4100 IF IN=50 THEN I$="INITIAL LOCATION IS FEASIBLE"
4200 PRINT I$ : PRINT : PRINT
4300 RETURN
4310 REM
4320 REM
4330 REM
4400 REM  DISTANCE(TA,PA)
4410 REM  DISTANCE CALCULATIONS
4500 TC=T0 : PC=PO
4510 IF ABS(TC-TA)>PI/2 THEN GOSUB 19000 : REM  SPECIAL CASE
4520 IF ABS(TC-TA)>PI/2 THEN GOTO 4700
4600 GOSUB 05500 : REM  CALC-LENGTH AL(TH1,PH1,TH2,PH2)
4700 D1=AL : TC=TD : PC=PD
4710 IF ABS(TC-TA)>PI/2 THEN GOSUB 19000 : REM  SPECIAL CASE
4720 IF ABS(TC-TA)>PI/2 THEN GOTO 4900
4800 GOSUB 05500 : REM  CALC-LENGTH
4900 D2=AL : TC=TB : PC=PB
4910 IF ABS(TC-TA)>PI/2 THEN GOSUB 19000 : REM  SPECIAL CASE
4920 IF ABS(TC-TA)>PI/2 THEN GOTO 5300
5200 GOSUB 05500 : REM  CALC-LENGTH
5300 D3=AL
5310 TA=T0 : PA=PO : TC=TD : PC=PD
5315 IF ABS(TC-TA)>PI/2 THEN GOSUB 19000 : REM  SPECIAL CASE
5320 IF ABS(TC-TA)>PI/2 THEN GOTO 5340
5330 GOSUB 05500 : REM  CALC-LENGTH
5340 O9=AL : TA=TH : PA=PH : TC=TB : PC=PB
5350 REM  D1=DISTANCE FROM ORIGIN TO POINT
5360 REM  D2=DISTANCE FROM DESTINATION TO POINT
5370 REM  D3=DISTANCE FROM TANKER BASE TO POINT
5380 REM  O9=DISTANCE FROM ORIGIN TO DESTINATION
5400 RETURN
5410 REM
5420 REM
5430 REM
5500 REM  CALC-LENGTH AL(TH1,PH1,TH2,PH2)
5600 REM  ALPHA=ANGLE BETWEEN THE TWO POINTS
5700 REM  K1=COSINE OF ALPHA
5800 REM  K3=TANGENT OF ALPHA
5900 S1=SIN(PC) : S2=SIN(PA) : D=TC-TA : CD=COS(D) : C1=COS(PC) : C2=COS(P
6000 K1=S1*S2*CD+C1*C2 : K2=K1↑2 : K5=SQR(1.-K2)
6100 K3=K5/K1 : AR=ATN(K3) : AL=AR*R
6150 REM  AL=ACTUAL LENGTH(DISTANCE BETWEEN THE TWO POINTS)
6200 RETURN
6210 REM
6220 REM
6230 REM
6300 REM  RANGE
6310 REM  CALCULATE MAXIMUM RANGE FOR EACH AIRCRAFT
6320 REM  WHEN LOADED WITH MAXIMUM FUEL
6400 GR=GM : WR=WO
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6500 GOSUB 07200 : REM  RT(GMAX,WO)
6600 R1=RT : GR=GC
6700 GOSUB 07200 : REM  RT(GCAP,WO)
6800 R2=RT : GR=HM
6900 GOSUB 07200 : REM  RT(HMAX,WO)
7000 R3=0.5*RT
7010 REM  R3=TANKER RANGE TAKING RETURN TRIP INTO ACCOUNT
7100 RETURN
7110 REM
7120 REM
7130 REM
7200 REM  RT(GO,WO)
7210 REM  CALCULATES RANGE FOR AIRCRAFT GIVEN ALLOWABLE FUEL AND CARGO WEIGHT
7220 REM  ASSUMES TRANSPORT AND TANKER HAVE SAME PERFORMANCE CURVES
7300 RT=(AP+A1*WR+A1/2*GR)*GR
7400 RETURN
7410 REM
7420 REM
7430 REM
7500 REM  SOLVE SUBPROBLEM ONE
7510 REM  FIND THE LEAST COMBINED FUEL FOR TRANSPORT AND TANKER
7520 REM  UNDER THE GIVEN CONDITIONS
7530 GOSUB 25400 : REM  FEASIBLE
7540 REM   UPDATE DISTANCES AFTER SEARCH IN SUBPROBLEM TWO
7600 SA=SQR(A↑2+2*A1*D2) : SB=SQR(AP↑2+2*A1*D3)
7700 C=SA+SB : WL=WO : DL=D1
7710 REM  C=CONSTANT USED IN LATER CALCULATIONS
7800 GOSUB 08200 : REM  FNTR(WL,DL)
7810 REM  FNTR=FUEL NEEDED BY TRANSPORT TO TRAVEL TO INITIAL POINT GIVEN
7900 LO=FT : HI=GM
8000 GOSUB 08600 : REM  SP1SEARCH(C)
8010 REM  LOOK FOR LEAST COMBINED FUEL COST
8100 RETURN
8110 REM
8200 REM  FNTR(WL,DL)(WO,D)
8210 REM  FNTR=FUEL NEEDED BY TRANSPORT TO REACH REFUEL POINT
8220 REM
8230 REM
8300 SC=(AP+A1*WL)↑2+2*A1*DL
8400 SD=SQR(SC) : FT=-WL-AP/A1+SD/A1
8500 RETURN
8510 REM
8520 REM
8530 REM
8600 REM  SP1SEARCH(C)
8610 REM  THIS IS A GOLDEN SECTION SEARCH ALONG THE FUEL RANGE FROM JUST BAREL
8620 REM  REACHING THE POINT TO THE MAXIMUM THE TRANSPORT CAN CARRY AT TAKEOFF
8700 SE=SQR(5.) : TU=(-1.+SE)*.5 : AC=0.00010
8800 G=LO : DD=HI
8900 XA=LO+(1.-TU)*(HI-LO)
9000 XB=LO+TU*(HI-LO)
9100 TX=XA
9200 GOSUB 11100 : REM  TFC1(XA,C)
9300 FA=T1 : TX=XB
9400 GOSUB 11100 : REM  TFC1(XB,C)
9500 FB=T1
9600 IF FA>FB THEN GOTO 09700
9650 IF FA<=FB THEN GOTO 10000
9700 G=XA : XA=XB : XB=G+TU*(DD-G) : FA=FB : TX=XB
9800 GOSUB 11100 : REM  TFC1(XB,C)
9900 FB=T1 : GOTO 10300
10000 DD=XB : XB=XA : XA=G+(1.-TU)*(DD-G) : FB=FA : TX=XA
10100 GOSUB 11100 : REM TFC1(XA,C)
10200 FA=T1
10300 IF (DD-G-AC)<=0 THEN GOTO 10400 : 10350 IF(DD-G-AC)>0 THEN GOTO 09600
10400 GA=XB : SF=(A+A1*XB)↑2-2*A1*D1
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10500 SG=SQR(SF) : SH=(C-SG)↑2+2*A1*D3 : SI=SQR(SH)
10600 HO=SI/A1-AP/A1 : F=FB
10700 IF FB<FA THEN GOTO 11000
10800 G0=XA : SF=(A+A1*XA)↑2-2*A1*D1 : SG=SQR(SF)
10900 SH=(C-SG)↑2+2*A1*D3 : SI=SQR(SH) : HO=SI/A1-AP/A1 : F=FA
10910 REM  F=LOWEST COMBINED FUEL COST FOR THE TRIAL POINT
11000 RETURN
11010 REM
11020 REM
11030 REM
11100 REM  TFC1(GO,C)
11110 REM  TFC1=TOTAL FUEL COST FOR SUBPROBLEM ONE
11120 REM  GIVEN INITIAL FUEL G0 AND CONSTANT C
11200 SF=(A+A1*TX)↑2-2*A1*D1
11300 SG=SQR(SF) : SH=(C-SG)↑2+2*A1*D3 : SI=SQR(SH)
11400 HO=SI/A1-AP/A1
11500 T1=TX+HO
11510 REM  T1=TFC1
11600 RETURN
11610 REM
11620 REM
11630 REM
11700 REM  SOLVE SUBPROBLEM TWO
11710 REM  SEARCHES FOR THE BEST REFUELLING LOCATION GIVEN INITIAL FUEL G0
11720 REM  STARTS FROM ORIGINAL POINT AND PROCEEDS TO CHECK DIAGONALS FOR
11730 REM  IMPROVEMENT
11740 REM  CHECKS EACH TRIAL POINT FOR FEASIBILITY BEFORE EVALUATING
11750 REM  IN CASE OF TIES THE AMOUNT OF CHANGE BETWEEN POINTS IS REDUCED
11760 REM  UNTIL THE TIE IS BROKEN OR THE ACCURACY LIMIT IS REACHED.
11800 DY=0.1 : T9=TH : P9=PH : DZ=0.1
11850 TA=TH : PA=PH
11900 GOSUB 22600 : REM  FVSP2
12000 FF=F2 : EP=0.1
12100 TH=T9+DZ : PH=P9+DZ : TY=0.
12200 GOSUB 25400 : REM  FEASIBLE-NOPRNT
12300 IF IN<>50 THEN GOTO 13400
12400 GOSUB 22600 : REM  FVSP2
12500 IF F2<FF THEN T9=TH
12600 IF F2<FF THEN P9=PH
12700 IF F2=FF THEN TY=1.
12800 IF F2<FF THEN FF=F2
12900 IF (FF=F2) AND (TY=0.) THEN GOTO 12100
13000 IF (FF=F2) AND (TY=1.)THEN DZ=DZ↑2
13100 IF DZ<E2 THEN GOTO 13300
13200 IF (FF=F2) AND (TY=1.) THEN GOTO 12100
13300 DZ=EP
13400 TH=T9-DZ : PH=P9+DZ : TY=0
13500 GOSUB 25400 : REM  FEASIBLE-NOPRNT
13600 IF IN<>50 THEN GOTO 14700
13700 GOSUB 22600 : REM  FVSP2
13800 IF F2<FF THEN T9=TH
13900 IF F2<FF THEN P9=PH
14000 IF F2=FF THEN TY=1.
14100 IF F2<FF THEN FF=F2
14200 IF (FF=F2) AND (TY=0.) THEN GOTO 13400
14300 IF (FF=F2) AND (TY=1.) THEN DZ=DZ↑2
14400 IF DZ<E2 THEN GOTO 14600
14500 IF (FF=F2) AND (TY=1.) THEN GOTO 13400
14600 DZ=EP
14700 TH=T9-DZ : PH=P9-DZ : TY=0.
14800 GOSUB 25400 : REM  FEASIBLE-NOPRNT
14900 IF IN<>50 THEN GOTO 16000
15000 GOSUB 22600 : REM  FVSP2
15100 IF F2<FF THEN T9=TH
15200 IF F2<FF THEN P9=PH
15300 IF F2=FF THEN TY=1
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15400 IF F2<FF THEN FF=F2
15500 IF (FF=F2) AND (TY=0.) THEN GOTO 14700
15600 IF (FF=F2) AND (TY=1.) THEN DZ=DZ↑2
15700 IF DZ<E2 THEN GOTO 15900
15800 IF (FF=F2) AND (TY=1.) THEN GOTO 14700
15900 DZ=EP
16000 TH=T9+DZ : PH=P9-DZ : TY=0.
16100 GOSUB 25400 : REM  FEASIBLE-NOPRNT
16200 IF IN<>50 THEN GOTO 17300
16300 GOSUB 22600 : REM  FVSP2
16400 IF F2<FF THEN T9=TH
16500 IF F2<FF THEN P9=PH
16600 IF F2=FF THEN TY=1.
16700 IF F2<FF THEN FF=F2
16800 IF (FF=F2) AND (TY=0.) THEN GOTO 16000
16900 IF (FF=F2) AND (TY=1.) THEN DZ=DZ↑2
17000 IF DZ<E2 THEN GOTO 17200
17100 IF (FF=F2) AND (TY=1.) THEN GOTO 16000
17200 DZ=EP
17300 IF EP<E2 THEN GOTO 17550
17400 EP=EP*DY : DZ=EP
17530 GOTO 12100
17550 TH=T9 : PH=P9
17560 GOSUB 50500 : REM  CONVERT SOLUTION TO LONG AND LAT TO SHOW ITERATIONS
17570 PRINT "SUBPROBLEM TWO TRIAL SOLUTION"
17575 PRINT L9;D7$,H8;D8$,F2*1000
17590 REM  PRINT SOMETHING TO SHOW OPERATOR THAT THE PROGRAM IS PROGRESSING
17600 RETURN
17610 REM
17620 REM
17630 REM
18000 REM  SOLVE SPECIAL CASE
18010 REM  REFUELLING IS NOT NEEDED HERE.
18020 REM  THEREFORE THE FUEL NEEDED FUNCTION IS SOLVED AND LOCATION FOR
18030 REM  REFUELLING IS SET TO 0
18100 SC=(AP+A1*W0)↑2+2*A1*09
18200 SD=SQR(SC) : G0=-W0-AP/A1+SD/A1
18300 L9=L2 : D7$=SL$ : H8=H2 : D8$=SM$ : H0=0 : FT=0 : F=G0 : K=1
18400 RETURN
18410 REM
18420 REM
18430 REM
19000 REM  SPECIAL CASE WHERE DISTANCE > PI/2
19100 AM=TC : PM=PC : AN=TA : PN=PA
19200 IF TC-TA<0 THEN TC=AM+(ABS(AM-AN))/2
19300 IF TC-TA>0 THEN TC=AM-(AM-AN)/2
19400 PC=(PM+PN)/2
19500 GOSUB 20000 : REM  CALC-LENGTH
19600 LQ=AQ : TA=TC : PA=PC : TC=AM : PC=PM
19700 GOSUB 20000 : REM  CALC-LENGTH
19800 LP=AQ : AL=LQ+LP : TC=AM : PC=PM : TA=AN : PA=PN
19900 RETURN
19910 REM
19920 REM
19930 REM
20000 REM  CALC-LENGTH FOR DISTANCE > PI/2 ROUTINE
20100 S1=SIN(PC) : S2=SIN(PA) : D=TC-TA : CD=COS(D) : C1=COS(PC) : C2=COS(PA)
20200 K1=S1*S2*CD+C1*C2 : K2=K1↑2 : K5=SQR(1.-K2)
20300 K3=K5/K1 : AR=ATN(K3) : AQ=AR*R
20400 RETURN
20410 REM
20420 REM
20430 REM
22600 REM  FVSP2(THSP2,PHSP2)
22610 REM  FVSP2=FUEL COST FUNCTION FOR SUBPROBLEM TWO
22620 REM  WHICH IS SLIGHTLY DIFFERENT FROM SUBPROBLEM ONE BECAUSE OF
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22630 REM  THE CONSTRAINTS BEING RELAXED.
22700 GOSUB 04400 : REM  DISTANCE
22710 REM  NEW DISTANCES MUST BE CALCULATED EVERY TIME THE REFUELLING POINT I
22720 REM  CHANGED.
22800 GOSUB 24600 : REM  FCTR(THSP2)
22810 REM  FCTR=FUEL COST FOR THE TRANSPORT-AGAING DIFFERENT BECAUSE OF RELA
22820 REM  CONSTRAINTS.
22900 WP=WO+(G0-FX) : WL=0.0 : DL=D3
23000 GOSUB 25000 : REM  FNTA(0,D3)
23050 REM  FUEL NEEDED BY TANKER TO RETURN AFTER REFUELLING
23100 F3=FY
23200 WL=WO : DL=D2
23300 GOSUB 08200 : REM  FNTR(WO,D2)
23350 REM  FUEL NEEDED BY TRANSPORT TO GET TO DESTINATION AFTER REFUELLING
23400 F4=FT
23500 W=F3+F4-(G0-FX)
23550 REM  NET REQUIREMENTS OF TANKER TO CARRY AS CARGO
23600 WL=WP : DL=D1
23700 GOSUB 08200 : REM  FNTR(WP,D1)
23750 REM  FUEL NEEDED BY TRANSPORT TO GET TO POINT WITH CARGO AND UNUSED FUE
23800 F5=FT : WL=W : DL=D3
24100 GOSUB 25000 : REM  FNTA(W,D3)
24110 REM  FUEL NEEDED BY TANKER TO GET TO REFUELLING POINT WITH NET FUEL(W)
24200 F7=FY
24400 F2=F5+F4+F7+F3
24410 REM  F2=FUEL COST FOR SUBPROBLEM TWO
24500 RETURN
24510 REM
24520 REM
24530 REM
24600 REM  FCTR(THSP2)
24610 REM FCTR=FUEL COST FOR TRANSPORT
24700 SV=(AP+A1*(WO+G0)) : SW=SV↑2-2*A1*D1
24800 SX=SQR(SW) : FX=G0+WO+AP/A1-SX/A1
24900 RETURN
24910 REM
24920 REM
24930 REM
25000 REM  FNTA(WL,DL)(WO,D)
25010 REM FNTA=FUEL NEEDED BY THE TANKER
25020 REM  ASSUMES TANKER AND TRANSPORT HAVE IDENTICAL PERFORMANCE CURVES.
25100 SC=(AP+A1*WL)↑2+2*A1*DL
25200 SD=SQR(SC) : FY=-WL-AP/A1+SD/A1
25300 RETURN
25310 REM
25320 REM
25330 REM
25400 REM  FEASIBLE-NOPRINT
25410 REM  SAME AS PREVIOUS ROUTINE BUT DOES NOT PRINT ANY MESSAGES.
25500 TA=TH : PA=PH
25600 GOSUB 04400 : REM  DISTANCE
25700 GOSUB 06300 : REM  RANGE
25710 WL=WO : DL=D1 : GOSUB 08200 : REM  FNTR(WO,DL)
25800 IN=0
25900 IF (D1>R1) OR (FT>GM) THEN IN=10
26000 IF D2>R2 THEN IN=20
26100 IF D3>R3 THEN IN=30
26200 IF IN<>10 AND IN<>20 AND IN<>30 THEN IN=50
26300 RETURN
26310 REM
26320 REM
26330 REM
28000 REM  CONVERSION TO RADIANS FOR COORDINATES
28010 REM  ALL COORDINATES ARE DEFINED IN RELATION TO THE ORIGIN.
28020 REM  THIS ALLOWS THE CATEGORIZATION OF MOST PROBLEMS.
28030 REM  THIS ROUTINE MUST BE EXPANDED TO ALLOW ROUTES ACROSS THE
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28040 REM  INTERNATIONAL DATE LINE OR ACROSS THE EQUATOR.  ALSO IF ROUTES ALO
28050 REM  THE SOUTHERN LATITUDES ARE TO BE CONSIDERED.
28100 L7=L1-L1
28200 IF (SJ$="W") AND (SL$="W") AND (SN$="W") AND (SQ$="W") THEN L4=L1-L2
28300 IF (SJ$="W") AND (SL$="W") AND (SN$="W") AND (SQ$="W") THEN L5=L1-L3
28400 IF (SJ$="W") AND (SL$="W") AND (SN$="W") AND (SQ$="W") THEN L6=L1-CA
28500 IF (SJ$="W") AND (SL$="W") AND (SN$="W") AND (SQ$="W") THEN QZ=1
28600 IF (QZ=1) AND (L1>L2) AND (L1>L3) AND (L1>CA) THEN QQ=1
28650 IF (QQ=1) THEN GOTO 50000
28700 IF (SJ$="W") AND (SL$="E") AND (SN$="W") AND (SQ$="W") THEN L4=L1+L2
28800 IF (SJ$="W") AND (SL$="E") AND (SN$="W") AND (SQ$="W") THEN L5=L1-L3
28900 IF (SJ$="W") AND (SL$="E") AND (SN$="W") AND (SQ$="W") THEN L6=L1-CA
29000 IF (SJ$="W") AND (SL$="E") AND (SN$="W") AND (SQ$="W") THEN QZ=2
29050 IF (QZ=2) AND (L4<=180) AND (L1>L3) AND (L1>CA) THEN QQ=2
29075 IF (QQ=2) THEN GOTO 50000
30000 IF (SJ$="W") AND (SL$="E") AND (SN$="W") AND (SQ$="E") THEN L4=L1+L2
30100 IF (SJ$="W") AND (SL$="E") AND (SN$="W") AND (SQ$="E") THEN L5=L1-L3
30200 IF (SJ$="W") AND (SL$="E") AND (SN$="W") AND (SQ$="E") THEN L6=L1+CA
30300 IF (SJ$="W") AND (SL$="E") AND (SN$="W") AND (SQ$="E") THEN QZ=3
30400 IF (QZ=3) AND (L4<=180) AND (L1>L3) AND (L6<=180) THEN QQ=3
30450 IF (QQ=3) THEN GOTO 50000
30500 IF (SJ$="W") AND (SL$="E") AND (SN$="E") AND (SQ$="W") THEN L4=L1+L2
30600 IF (SJ$="W") AND (SL$="E") AND (SN$="E") AND (SQ$="W") THEN L5=L1+L3
30700 IF (SJ$="W") AND (SL$="E") AND (SN$="E") AND (SQ$="W") THEN L6=L1-CA
30800 IF (SJ$="W") AND (SL$="E") AND (SN$="E") AND (SQ$="W") THEN QZ=4
30900 IF (QZ=4) AND (L4<=180) AND (L5<=180) AND (CA<L1) THEN QQ=4
30950 IF (QQ=4) THEN GOTO 50000
31000 IF (SJ$="W") AND (SL$="E") AND (SN$="E") AND (SQ$="E") THEN L4=L1+L2
31100 IF (SJ$="W") AND (SL$="E") AND (SN$="E") AND (SQ$="E") THEN L5=L1+L3
31200 IF (SJ$="W") AND (SL$="E") AND (SN$="E") AND (SQ$="E") THEN L6=L1+CA
31300 IF (SJ$="W") AND (SL$="E") AND (SN$="E") AND (SQ$="E") THEN QZ=5
31400 IF (QZ=5) AND (L4<=180) AND (L5<=180) AND (L6<=180) THEN QQ=5
31450 IF (QQ=5) THEN GOTO 50000
31500 IF (SJ$="E") AND (SL$="E") AND (SN$="E") AND (SQ$="E") THEN L4=L2-L1
31600 IF (SJ$="E") AND (SL$="E") AND (SN$="E") AND (SQ$="E") THEN L5=L3-L1
31700 IF (SJ$="E") AND (SL$="E") AND (SN$="E") AND (SQ$="E") THEN L6=CA-L1
31800 IF (SJ$="E") AND (SL$="E") AND (SN$="E") AND (SQ$="E") THEN QZ=6
31900 IF (QZ=6) AND (L2>L1) AND (L3>L1) AND (CA>L1) THEN QQ=6
31950 IF (QQ=6) THEN GOTO 50000
33000 IF (SJ$="W") AND (SL$="W") AND (SN$="W") AND (SQ$="W") THEN L4=L2-L1
33100 IF (SJ$="W") AND (SL$="W") AND (SN$="W") AND (SQ$="W") THEN L5=L3-L1
33200 IF (SJ$="W") AND (SL$="W") AND (SN$="W") AND (SQ$="W") THEN L6=CA-L1
33300 IF (SJ$="W") AND (SL$="W") AND (SN$="W") AND (SQ$="W") THEN QZ=7
33400 IF (QZ=7) AND (L2>L1) AND (L3>L1) AND (CA>L1) THEN QQ=7
33450 IF (QQ=7) THEN GOTO 50000
33500 IF (SJ$="E") AND (SL$="W") AND (SN$="W") AND (SQ$="W") THEN L4=L1+L2
33600 IF (SJ$="E") AND (SL$="W") AND (SN$="W") AND (SQ$="W") THEN L5=L1+L3
33700 IF (SJ$="E") AND (SL$="W") AND (SN$="W") AND (SQ$="W") THEN L6=L1+CA
33800 IF (SJ$="E") AND (SL$="W") AND (SN$="W") AND (SQ$="W") THEN QZ=8
33900 IF (QZ=8) AND (L4<=180) AND (L5<=180) AND (L6<=180) THEN QQ=8
33950 IF (QQ=8) THEN GOTO 50000
34000 IF (SJ$="E") AND (SL$="W") AND (SN$="W") AND (SQ$="E") THEN L4=L1+L2
34100 IF (SJ$="E") AND (SL$="W") AND (SN$="W") AND (SQ$="E") THEN L5=L1+L3
34200 IF (SJ$="E") AND (SL$="W") AND (SN$="W") AND (SQ$="E") THEN L6=L1-CA
34300 IF (SJ$="E") AND (SL$="W") AND (SN$="W") AND (SQ$="E") THEN QZ=9
34400 IF (QZ=9) AND (L4<=180) AND (L5<=180) AND (CA<L1) THEN QQ=9
34450 IF (QQ=9) THEN GOTO 50000
34500 IF (SJ$="E") AND (SL$="W") AND (SN$="E") AND (SQ$="W") THEN L4=L1+L2
34600 IF (SJ$="E") AND (SL$="W") AND (SN$="E") AND (SQ$="W") THEN L5=L1-L3
34700 IF (SJ$="E") AND (SL$="W") AND (SN$="E") AND (SQ$="W") THEN L6=L1+CA
34800 IF (SJ$="E") AND (SL$="W") AND (SN$="E") AND (SQ$="W") THEN QZ=10
34900 IF (QZ=10) AND (L4<=180) AND (L3<L1) AND (L6<=180) THEN QQ=10
34950 IF (QQ=10) THEN GOTO 50000
35000 IF (SJ$="E") AND (SL$="W") AND (SN$="E") AND (SQ$="E") THEN L4=L1+L2
35100 IF (SJ$="E") AND (SL$="W") AND (SN$="E") AND (SQ$="E") THEN L5=L1-L3
35200 IF (SJ$="E") AND (SL$="W") AND (SN$="E") AND (SQ$="E") THEN L6=L1-CA
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35300 IF (SJ$="E") AND (SL$="W") AND (SN$="E") AND (SQ$="E") THEN QZ=11
35400 IF (QZ=11) AND (L4<=180) AND (L3<L1) AND (CA<L1) THEN QQ=11
35450 IF (QQ=11) THEN GOTO 50000
35500 IF (SJ$="E") AND (SL$="E") AND (SN$="E") AND (SQ$="E") THEN L4=L1-L2
35600 IF (SJ$="E") AND (SL$="E") AND (SN$="E") AND (SQ$="E") THEN L5=L1-L3
35700 IF (SJ$="E") AND (SL$="E") AND (SN$="E") AND (SQ$="E") THEN L6=L1-CA
35800 IF (SJ$="E") AND (SL$="E") AND (SN$="E") AND (SQ$="E") THEN QZ=12
35900 IF (QZ=12) AND (L2<L1) AND (L3<L1) AND (CA<L1) THEN QQ=12
35950 IF (QQ=12) THEN GOTO 50000
50000 T0=L7 : P0=PI/2-H1*PI/180
50100 TD=L4*PI/180 : PD=PI/2-H2*PI/180
50200 TB=L5*PI/180 : PB=PI/2-H3*PI/180
50300 TH=L6*PI/180 : PH=PI/2-CB*PI/180
50310 REM  CONVERT THE COORDINATES TO RADIANS
50320 REM
50330 REM
50340 REM
50350 RETURN
50500 REM  CONVERT SOLUTION TO LONGITUDE AND LATITUDE
50510 REM  THIS ROUTINE WOULD ALSO NEED TO BE EXPANDED IF MORE CASES ARE TO B
50520 REM  CONSIDERED.
50600 H8=90-PH*180/PI : D8$="N"
50700 IF (QQ<=5) OR ((QQ>=8) AND (QQ<=12)) THEN L9=L1-(TH*180/PI)
50800 IF (QQ=6) OR (QQ=7) THEN L9=L1+(TH*180/PI)
50900 IF (QQ<=5) AND (L9>=0) THEN D7$="W"
51000 IF (QQ<=5) AND (L9<0) THEN D7$="E"
51100 IF (QQ<=5) AND (L9<0) THEN L9=ABS(L9)
51200 IF QQ=6 THEN D7$="E"
51300 IF QQ=7 THEN D7$="W"
51400 IF ((QQ>=8) AND (QQ<=11)) AND (L9>=0) THEN D7$="E"
51500 IF ((QQ>=8) AND (QQ<=11)) AND (L9<0) THEN D7$="W"
51600 IF ((QQ>=8) AND (QQ<=11)) AND (L9<0) THEN L9=ABS(L9)
51700 IF QQ=12 THEN D7$="E"
51800 RETURN
51810 REM
51820 REM
51830 REM
60500 REM  PRINT MAP OF U.S. AND EUROPE
60501 PRINT "J"
60507 POKE1188,43:POKE1191,43:POKE1194,43:POKE1197,43:POKE1200,43:POKE1284,4
60508 POKE1285,43:POKE1286,43:POKE1212,43:POKE1215,43:POKE1227,43:POKE1228,4
60509 POKE1231,43:POKE1232,43:POKE1233,43:POKE1234,43:POKE1238,43:POKE1240,4
60510 POKE1251,43:POKE1254,43:POKE1255,43:POKE1267,43:POKE1271,43:POKE127
60511 POKE1278,43:POKE1279,43:POKE1291,43:POKE1294,43:POKE1295,43:POKE130
60512 POKE1311,43:POKE1313,43:POKE1314,43:POKE1315,43:POKE1328,43:POKE1329,4
60513 POKE1332,43:POKE1333,43:POKE1334,43:POKE1335,43:POKE1348,43:POKE1349,4
60514 POKE1351,43:POKE1355,43:POKE1368,43:POKE1370,43:POKE1372,43:POKE137
60515 POKE1374,43:POKE1390,43:POKE1391,43:POKE1396,43:POKE1407,43:POKE1410,4
60516 POKE1411,43:POKE1430,43:POKE1436,43:POKE1448,43:POKE1449,43:POKE1450,
60517 POKE1477,43:POKE1489,43:POKE1515,43:POKE1513,43:POKE1514,43:POKE1516,4
60518 POKE1528,43:POKE1553,43:POKE1567,43:POKE1573,43:POKE1574,43:POKE1574,4
60519 POKE1575,43:POKE1592,43:POKE1605,43:POKE1607,43:POKE1608,43:POKE1609,4
60520 POKE1610,43:POKE1611,43:POKE1612,43:POKE1614,43:POKE1615,43:POKE1616,4
60521 POKE1617,43:POKE1618,43:POKE1631,43:POKE1648,43:POKE1653,43:POKE1655,4
60522 POKE1658,43:POKE1670,43:POKE1687,43:POKE1694,43:POKE1696,43:POKE169 ,4
60523 POKE1698,43:POKE1702,43:POKE1706,43:POKE1707,43:POKE1708,43:POKE1709,4
60524 POKE1710,43:POKE1727,43:POKE1738,43:POKE1739,43:POKE1742,43:POKE1742,4
60525 POKE1743,43:POKE1744,43:POKE1746,43:POKE1750,43:POKE1751,43:POKE1766,4
60526 POKE 1779,43:POKE1785,43:POKE1787,43:POKE1788,43:POKE1789,43:POKE1791,4
60527 POKE1805,43:POKE1819,43:POKE1820,43:POKE1826,43:POKE1827,43:POKE1828,4
60528 POKE1829,43:POKE1830,43:POKE1845,43:POKE1860,43:POKE1861,43:POKE1862,4
60529 POKE1863,43:POKE1869,43:POKE1870,43:POKE1872,43:POKE1873,43:POKE1874,4
60530 POKE1886,43:POKE1901,43:POKE1902,43:POKE1910,43:POKE1911,43:POKE1916,4
60531 POKE1926,43:POKE1927,43:POKE1928,43:POKE1929,43:POKE1930,43:POKE1942,4
60532 POKE 1951,43:POKE1957,43:POKE1958,43:POKE1971,43:POKE1972,43:POKE1981,4
60533 POKE1991,43:POKE2000,43:POKE2012,43:POKE2021,43
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60534 FOR U=1992TO1999:POKEU,102:NEXTU:FOR U=2013TO2020:POKEU,102:NEXTU
60535 FOR U=1952TO1956:POKEU,102:NEXTU:FOR U=1973TO1980:POKEU,102:NEXTU
60536 FOR U=1912TO1915:POKEU,102:NEXTU:FOR U=1931TO1941:POKEU,102:NEXTU
60537 FOR U=1886TO1900:POKEU,102:NEXTU:FOR U=1846TO1859:POKEU,102:NEXTU
60538 FOR U=1806TO1818:POKEU,102:NEXTU:FOR U=1821TO1823:POKEU,102:NEXTU
60539 FOR U=1767TO1778:POKEU,102:NEXTU:FOR U=1780TO1783:POKEU,102:NEXTU
60540 FOR U=1728TO1737:POKEU,102:NEXTU:FOR U=1664TO1669:POKEU,102:NEXTU
60541 FOR U=1688TO1693:POKEU,102:NEXTU:FOR U=1699TO1701:POKEU,102:NEXTU
60542 FOR U=1624TO1630:POKEU,102:NEXTU:FOR U=1649TO1652:POKEU,102:NEXTU
60543 FOR U=1659TO1663:POKEU,102:NEXTU:FOR U=1584TO1591:POKEU,102:NEXTU
60544 FOR U=1619TO1623:POKEU,102:NEXTU:FOR U=1544TO1552:POKEU,102:NEXTU
60545 FOR U=1576TO1583:POKEU,102:NEXTU:FOR U=1504TO1512:POKEU,102:NEXTU
60546 FOR U=1529TO1543:POKEU,102:NEXTU:FOR U=1464TO1476:POKEU,102:NEXTU
60547 FOR U=1490TO1503:POKEU,102:NEXTU:FOR U=1424TO1429:POKEU,102:NEXTU
60548 FOR U=1431TO1435:POKEU,102:NEXTU:FOR U=1451TO1463:POKEU,102:NEXTU
60549 FOR U=1384TO1389:POKEU,102:NEXTU:FOR U=1392TO1395:POKEU,102:NEXTU
60550 FOR U=1412TO1423:POKEU,102:NEXTU:FOR U=1375TO1383:POKEU,102:NEXTU
60551 FOR U=1336TO1343:POKEU,102:NEXTU:FOR U=1296TO1303:POKEU,102:NEXTU
60552 FOR U=1256TO1263:POKEU,102:NEXTU:FOR U=1216TO1223:POKEU,102:NEXTU
60553 FOR U=1184TO1187:POKEU,102:NEXTU:FOR U=1224TO1226:POKEU,102:NEXTU
60554 FOR U=1264TO1266:POKEU,102:NEXTU:FOR U=1304TO1306:POKEU,102:NEXTU
60555 FOR U=1344TO1347:POKEU,102:NEXTU:FOR U=1352TO1354:POKEU,102:NEXTU
60556 FOR U=1192TO1193:POKEU,102:NEXTU:FOR U=1198TO1199:POKEU,102:NEXTU
60557 FOR U=1213TO1214:POKEU,102:NEXTU:FOR U=1292TO1293:POKEU,102:NEXTU
60558 FOR U=1408TO1409:POKEU,102:NEXTU:FOR U=1568TO1569:POKEU,102:NEXTU
60559 FOR U=1704TO1705:POKEU,102:NEXTU:FOR U=1740TO1741:POKEU,102:NEXTU
60560 POKE1239,102:POKE1252,102:POKE1253,102:POKE1272,102:POKE1312,102
60561 POKE1369,102:POKE1703,102:POKE1745,102:POKE1786,102
60600 V=24-INT(H1/3.5)
60605 IF SJ$="W" THEN H=27-INT(L1/4)
60610 IF SJ$="E" THEN H=27+INT(L1/4)
60615 POKE 1024+H+40*V,15
60620 POKE 55296+H+40*V,0
60625 V=24-INT(H2/3.5)
60630 IF SL$="W" THEN H=27-INT(L2/4)
60635 IF SL$="E" THEN H=27+INT(L2/4)
60640 POKE 1024+H+40*V,4
60645 POKE 55296+H+40*V,0
60650 V=24-INT(H3/3.5)
60655 IF SN$="W" THEN H=27-INT(L3/4)
60660 IF SN$="E" THEN H=27+INT(L3/4)
60665 POKE 1024+H+40*V,2
60670 POKE 55296+H+40*V,0
60675 V=24-INT(H8/3.5)
60680 IF D7$="W" THEN H=27-INT(L9/4)
60685 IF D7$="E" THEN H=27+INT(L9/4)
60690 POKE 1024+H+40*V,18
60695 POKE 55296+H+40*V,1
60700 REM  PLACE COORDINATES ON MAP FOR ORIGIN,DEST.TANKER BASE, AND SOLUTI
60710 REM  POINT FOR REFUELLING
60720 STOP
60730 RETURN
61000 STOP
62000 END

READY.
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KEY FOR TRIAL RUNS CHART

LOCATION	LONGITUDE	LATITUDE	CODE
New Jersey	75W	40N	A
Delaware	75W	38N	B
North Carolina	78W	35N	C
Puerto Rico	66W	18N	D
Azores Islands	25W	37N	F
Iceland	20W	65N	F
Germany	10E	50N	G
Turkey	30E	40N	H
Saudi Arabia	47E	25N	I
Egypt	28E	30N	J
England	0E/W	52N	K

WEIGHT	CODE
100,000 lbs	1
200,000 lbs	2

FUEL ACCURACY

100 means to within 100 lbs

POSITION ACCURACY

1 means to within 1 degree

TRIAL RUNS

RUN NO	O	D	TB	INITIAL REFUEL	W	FUEL ACCR	POS ACCR	GO	TRANSFER	TANKER	TOTAL	OPTIMAL LOCATION	TIME	ITERATIONS
1	A	H	D	40W 35N	2	100	1	113,521	136,704	149,055	399,280	49W 40N	2:19	2
2	G	C	F	35W 40N	2	100	1	79,431	130,541	1,058	211,030	20W 65N	4:46	4
3	B	I	E	30W 30N	1	100	1	137,142	144,464	5866	287,472	25W 37N	3:03	2
4	B	J	D	30W 25N	2	100	1	118,712	152,513	156,783	428,008	42W 36N	1:53	2
5	C	K	F	30W 50N	2	100	1	129,882	54,425	19,269	203,576	28W 63N	2:40	2

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